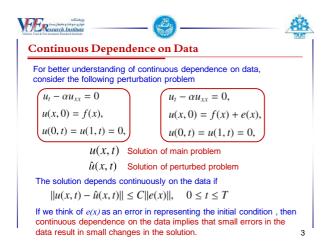
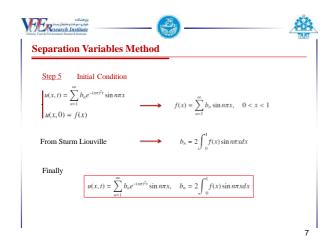


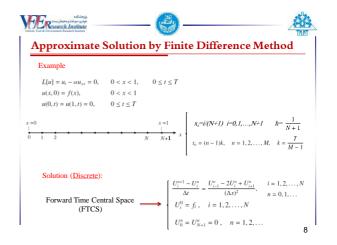
| The colu | ition is said to depend cou | ntinuously on the data function $f(x)$ |
|--------------|---|--|
| | | C independent of f such that |
| 1 | $ u(x,t) \le C f(x) , 0 \le C f(x) $ | $\leq t \leq T$ |
| de | enotes a general form tha | t could be |
| L^{∞} | the maximum norm | $ f(x) = \max f(x) , 0 \le x \le 1$ |
| | or the energy norm | $ f(x) = \left\{ \int_0^1 f(x)^2 dx \right\}^{\frac{1}{2}}$ |

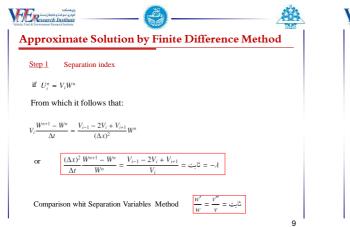


| Example | | |
|---------------------------|--|--|
| $L[u] = u_t - \alpha u_t$ | $\alpha_{x} = 0, 0 < x < 1, 0 \le t \le T \alpha = 1$ | |
| u(x,0)=f(x), | 0 < x < 1 | |
| u(0,t)=u(1,t) | $= 0, 0 \le t \le T$ | |
| | ration Variables Method $u(x, t) = v(x)w(t)$ | |
| then | ,, | |
| $u_l = vw'$, u | $t_{xx} = v'' w$ | |
| thus | vw' - v''w = 0 | |
| | vw' - v''w = 0 $\frac{w'}{w} = \frac{v''}{v} = \frac{v''}{v}$ | |
| | | |

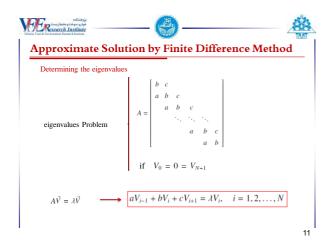
| Kenter and Andread And | |
|--|--|
| Separation Variables Method | Separation Variables Method |
| Step 2 Boundary Conditions | <u>Step 3</u> Determining the transient behavior |
| $v(0) = v(1) = 0$ $v \neq 0$ $v'' = -\lambda^2 v, 0 < x < 1, v(0) = v(1) = 0$ | From step 1 and step 2 $\xrightarrow{w'} w' = -(n\pi)^2$, $n = 1, 2,$ Thus: $w_n(t) = b_n e^{-(n\pi)^2 t}$, $n = 1, 2,$ |
| From which it follows that: | |
| $\lambda_n^2 = (n\pi)^2, v_n(x) = \sin n\pi x, n = 1, 2, \dots$ | <u>Step 4</u> Linear combination $u(x,t) = v_n(x)w_n(t) = \sum_{n=1}^{\infty} h_n e^{-(n\pi)^2 t} \sin n\pi x$ |
| | |
| 1 | 5 6 |
| | |

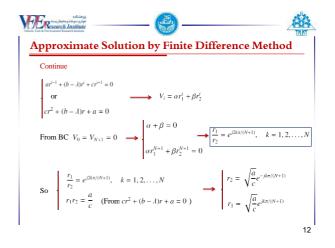


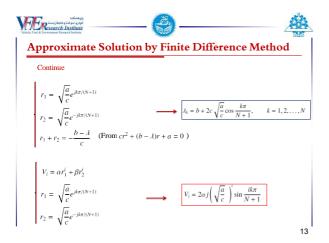


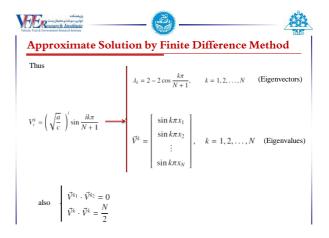


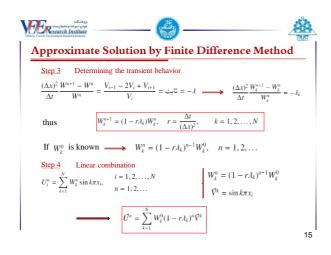
| ppro. | imate | Solutio | n by Fin | ite Diff | erence Method | |
|--|-----------------------------|------------------------------|-------------------|----------|--|----|
| Step 2 | | y Conditions | | | | |
| $V_0 = 0$ | $= V_{N+1}$ | | | | | |
| $\frac{(\Delta x)^2}{\Delta t} \frac{V}{\Delta t}$ | $\frac{W^{n+1} - W^n}{W^n}$ | $= \frac{V_{i-1} - 2V}{V_i}$ | بت = ⁽ | √ = −ئ | | |
| | | | i = 1, 2,, N | | = 0 | |
| or | | 2 - | 1 -1 1 2 -1 | | | |
| | _ | | | | | |
| $A\vec{V} = \lambda \vec{V}$ | $] \longrightarrow$ | A = | an an an Ar an | 5. 1 | $\vec{V} = [V_1, V_2, V_3, \dots, V_{N-1}, V_N]$ | JT |
| | | | | -1 2 -1 | | |
| | | | | -1 2 | _ | |











| The second secon | |
|--|----|
| Approximate Solution by Finite Difference Method | |
| <u>Step 5</u> Initial Condition | |
| Considering $f = [f_1, f_2,, f_N]^T$ and $U_i^0 = f_i$, $i = 1, 2,, N$ | |
| thus $f = \sum_{k=1}^{N} W_k^0 \vec{V}^k$ | |
| Orthogonally principle $W_k^0 = \frac{\vec{f} \cdot \vec{V}^k}{\vec{V}^k \cdot \vec{V}^k}, k = 1, 2, \dots, N$ | |
| Comparison whit Separation Variables Method $b_n = 2 \int_0^1 f(x) \sin n\pi x dx$ | |
| | 16 |

